Objective characterisation of fire regimes for science-based management of fire-prone landscapes.

Matthias Boer[1,2*], Rohan Sadler[2], Pauline Grierson[1,2]

[1]*Bushfire Cooperative Research Centre, East Melbourne 3002 VIC, Australia.* [2]*Ecosystems Research Group, School of Plant Biology M090, The University of Western Australia, 35 Stirling Highway, Crawley 6009 WA, Australia.*

* Corresponding author. Email: Matthias.Boer@uwa.edu.au

Abstract

In fire-prone landscapes many of the ecosystem properties that land managers aim to maintain or enhance are closely linked to the incidence and patterning of fire, particularly of fire intensity, extent, season, and frequency of burning. Characteristic combinations of these attributes are often referred to as fire regimes. The term fire regime is mostly used as a qualitative descriptor rather than a quantitative multivariate characterisation of fire in the landscape. A lack of methods to objectively measure and characterise spatiotemporal patterns of fire attributes is one of the factors impeding a more quantitative use of the fire regime concept. We have developed and trialled a new non-parametric method to quantify the similarity among fire histories at multiple sites in the landscape. This method provides a stronger basis for disentangling complex relationships among fire attributes. The temporal pattern of fire incidence is but one aspect of a fire regime. For research and management, a more significant characterisation of the fire regime includes descriptors of burn severity. Consequently, we have also developed an empirical model for the remote sensing of burn severity and propose an approach to incorporate a measure of burn severity in the classification of fire regimes. Key elements of our approach were trialled in two areas in SW Western Australia (Warren Region and Perth Hills).

Introduction

Land management in general, and fire management in particular, needs increasingly to satisfy multiple and often conflicting objectives. Management must also be accountable and include public consultation, as well as adapt to constantly shifting political, economic or climate contexts. These changes require a strengthening of the scientific support and justification of management decisions. Strategic measures employed in fire management, such as prescribed burning, manipulate the "fire regime" in order to reduce the risk of loss of property, life or ecological values posed by wildfire. A fire regime is, in fact, a suite of attributes describing the variation inherent in the frequency, intensity, timing, location and extent of fires.

To date, characterisation of the fire regime has been difficult owing to a lack of long-term and landscape-scale data, uncertainty about the scale dependence of data, specificity of the ecosystem at hand, and the difficulty of modelling the often complex relationships between different elements of the fire regime. In this paper, we develop an approach to characterise fire regimes statistically using a nonparametric measure of the "inhibition" effect of one fire on another. This measure models the incidence of fire as a "point process", and is a first step towards further work in characterising fire regimes through studying interactions between multiple fire attributes. The benefit of such a general approach is that other variables, such as burn severity, may then be modelled through "marked" point processes, where the extent of "inhibition" is a function of the severity of the previous fire. The concept of inhibition has obvious extensions to prescribed burns, which are generally of lower severity than wildfire due to the controlled nature of their implementation. After introducing an "inhibition metric" and a method for mapping burn severity, we also discuss how marked point processes may be used to design a regime of prescribed burns to optimally meet management objectives.

Fire Occurrence as a Point Process

The classification of different historical patterns of fire incidence at separate locations into similar fire regimes is dogged by high variability in short-term patterns of fire incidence. This high variability between sites easily results from the "local" realisation of fire incidence when treated as a stochastic process, despite sites sharing similar long-term patterns or distributions of fire events. One approach is to model fire occurrence at a single location as a "point process", where the timing of fire events are conceived as points on a real time line (Fig. 1). The statistical theory of recurrent events as these point processes is well developed, and includes bias corrections for interval censoring when events are observed only over a limited time frame (Polakow and Dunne 1999; Cook and Lawless 2007). Fire histories have often been represented as point processes; for example, when described by the Weibull model (Clark 1979; McCarthy et al. 2001; Moritz et al. 2008). These models are useful in characterising fire occurrences by defining a renewal function: a local function assigning probabilities of fire occurrence based on fuel age (or time since last fire). However, we pursue nonparametric estimation of the renewal function (e.g. Pena et al. 2001; Markovich and Krieger 2006) as nonparametric estimates provide greater flexibility than a parametric model approach in not superimposing the assumptions behind a given model structure upon the data.

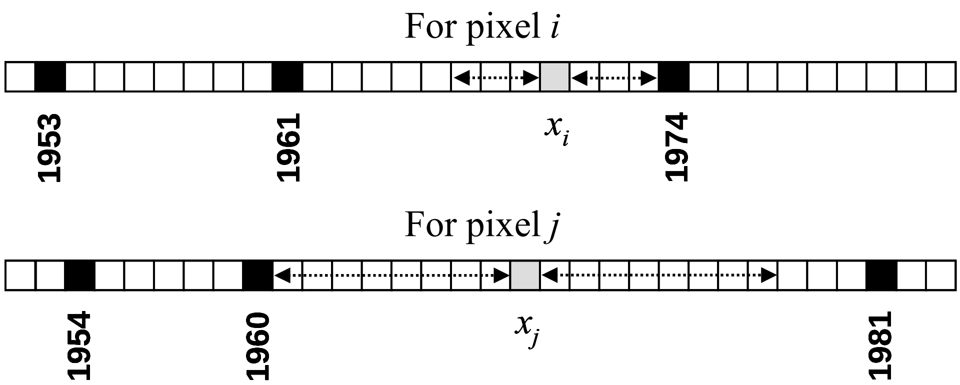

Figure 1. Schematic representation of the historical record of fire incidence at two locations i and j. Years in which a fire was recorded at the indicated location are shown in black and non-fire years in white. The dashed arrows indicate the bi-directional waiting times for one time point on each record, x_i and x_j (grey box). The waiting time is the distance in time between a specific point in time (x) and the nearest time of fire occurrence before or after that point in time. The nonparametric estimate of the empty space function $\hat{F}_i(t)$ is then the distribution of these waiting times.

The Inhibition Distance I_α

We expand the notion of a survival function of a process (i.e., the probability of no fire before time t) by introducing the empty space function $F(t)$ (or linear contact distribution), which treats time as bi-directional (i.e., considering a symmetrical time window centred on the time of observation, see Fig. 1). This bi-directionality increases the robustness of any attempt to characterise the temporal patterning of fires, pertinent when the total number of fires observed at an individual location i are few (i.e., the observation period is small compared to the period required to fully characterise the fire regime). The small observation window compared to the process at hand means heavy censoring of observations takes place: there are unobserved fires before and after the period of observation that will introduce large biases into standard estimators of the renewal function, such as the Nelson-Aalen. A bias-correction is available in the Kaplan-Meier estimator of $F(t)$ (Baddeley and Gill 1997):

$$\hat{F}_i(t) = 1 - \exp\left\{ - \int_W \frac{1\{w(x) \le c(x)\} 1\{w(x) \le t\}}{\left| W_{(-w(x))} \setminus \Phi_{(+w(x))} \right|} dx \right\} \qquad (1)$$

where $w(x)$ is the waiting time, the distance (in time) between a specific point x in time and the nearest time of fire occurrence (either before or after that point in time); $c(x)$ is the censoring distance, the minimum distance between a specific point and the boundary of the observational window; $1\{\ \}$ is the indicator function returning a value of 1 if the statement between the brackets is true, 0 if false; $W_{(-w(x))}$ is the observation window W minus a buffer zone of width $w(x)$; and $\Phi_{(+w(x))}$ is the union of balls of radius $w(x)$ centred in time upon the observed occurrence of fires described by the process Φ.

The number of fires observed during the period of record keeping at a location i is N_i. The patterning of fires at a location can be compared to a neutral (or random) model of fire occurrence generated by a Poisson process with intensity $\lambda_i = N_i / |W|$ (alternative models may also be employed such as the Weibull). Confidence envelopes for the $\hat{F}_i(t)$ estimate of the neutral model's linear contact distribution can be generated from B Monte Carlo simulations. The inhibition distance I_α (Fig. 2) for a given confidence level α is then defined as:

$$I_\alpha = \min\left\{ t : \hat{F}_i(t) = \hat{F}_{\lambda_i}^{(\alpha)}(t), t > 0 \right\} \qquad (2)$$

where $\hat{F}_{\lambda_i}^{(\alpha)}(t)$ is the $\lceil B\alpha \rceil^{th}$ envelope of the Kaplan-Meier estimate of the linear contact distribution, generated through Monte Carlo simulation of the Poisson process with intensity λ_i. The inhibition distance measures the extent to which individual events "repel" each other according to a single sided statistical test of significance $\alpha < 0.5$, i.e. a high inhibition distance means that fires are more evenly spaced out than for an equivalent random arrangement of fire occurrences. Such an interpretation is cogent in the context of fuel age effects, where recently burnt sites are unlikely to burn again due to the lack of regeneration of sufficient fuels over short time periods to carry a fire. Indeed, the inhibition distance can be interpreted as a measure of the strength of fuel age effects on fire incidence. Other properties of I_α include: (i) if the patterning of fires are equivalent to a random or clustered process then $I_\alpha = 0$; and, (ii) the strength of any clustering may be identified by taking $\alpha > 0.5$.

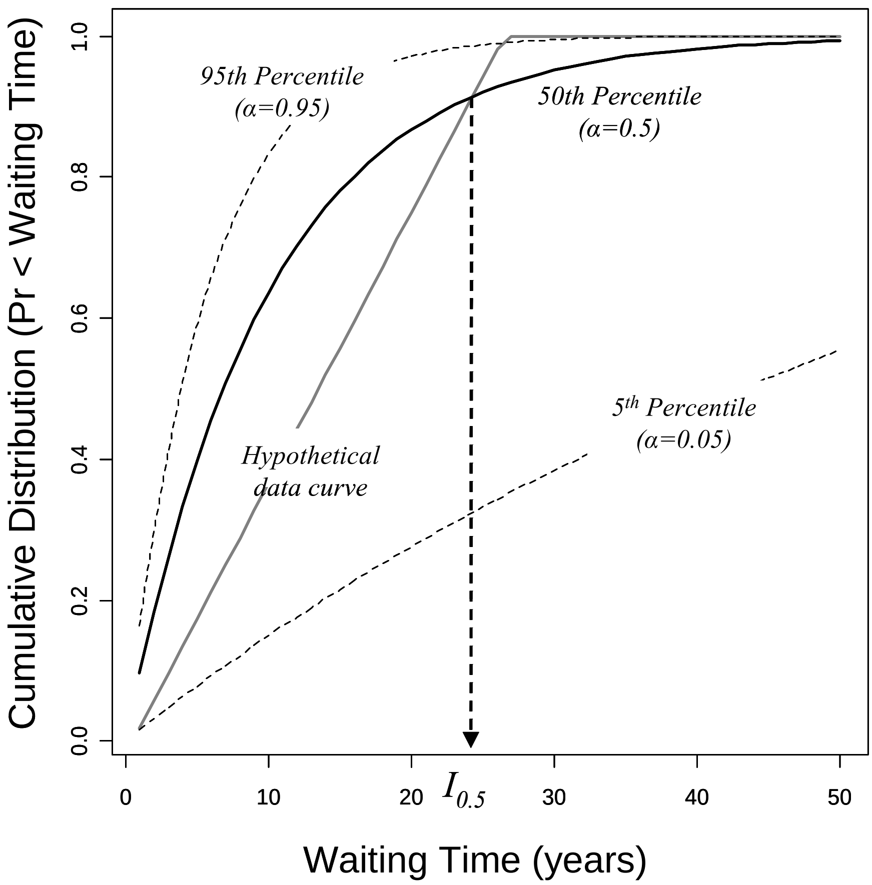

Figure 2. Cumulative distributions of the waiting times for a hypothetical location (continuous grey curve) and the neutral model (black curves) defined by the 5th, 50th and 95th percentiles of the one sided confidence bound (corresponding to different significance levels α). The inhibition distance $I_{.5}$=25 for the grey data curve is the waiting time defined by the first crossover point between the data curve and the 50th percentile confidence bound of the neutral model (dashed black arrow).

Characterisation of fire regimes in the Warren Region

The proposed method for the characterisation of fire regimes has been trialled in the Warren Region of SW Western Australia. The predominant forest type in the region is jarrah (*Eucalyptus marginata* Donn ex Sm.), usually mixed with marri (*Corymbia calophylla* R.Br.), while Karri (*Eucalyptus diversicolor* F. Muell.) forests are also widespread. Sedgelands and coastal heathlands are also well represented (Bradshaw et al. 1997). Most of the ~1.4 million hectare Warren Region has been managed by the State Government since 1919 with the formation of the Forests Department, and subsequently by the Department of Environment and Conservation (DEC) and its predecessor, the Department of Conservation and Land Management (CALM). The fire history of the Warren Region (1953-2004) has recently been compiled in a geographical information system (Hamilton et al., subm.), providing a unique opportunity to investigate spatiotemporal patterns of fire incidence, severity and impacts.

The fire history data base holds fire records in vector format. We firstly grouped the records by year and then converted all annual fire maps to raster format with 1 ha pixels. A 40 km by 40 km study area located in the centre of the Warren Region was selected for statistical analyses. Records of fire incidence (planned and unplanned fires) between 1953 and 2004 were extracted for all pixels producing an array of 52 (years) by 160,000 (pixels), with I_α computed at each cell. A significance level $\alpha = 0.5$ and $B = 500$ Monte Carlo simulations were chosen to compute $I_{0.5}$. A lower significance level would assign fewer locations a strictly positive I_α value, given the low number of fires observed at many locations in the fire history database.

In comparison to characterising locations by the number of fires only, the $I_{0.5}$ metric simplifies the picture: different regions with different number of observed fires can maintain a similar patterning of fires in comparison to a random pattern of fire occurrence (Fig. 3). The $I_{0.5}$ metric is strongly correlated with a number of geographical variables through a multivariate linear regression (Table 1). The interpretation of these correlations makes ecological “sense”. For example, a high topographic wetness index, computed from local slope angle and upslope drainage area (Beven and Kirkby 1979), is correlated with a strong inhibition distance: the higher the wetness index the smaller the seasonal window in which fuels are sufficiently dry to carry fire, thereby increasing the return time of fires. The key explanatory variable was vegetation type, explaining 45% of the variation to which the remaining variables explain only a further 6%. About 50% of the variation in $I_{0.5}$ values remains unexplained in the reported regression model. This is not surprising as the majority of the fires in the Warren Region were planned fires (Abbott and Burrows 2003).

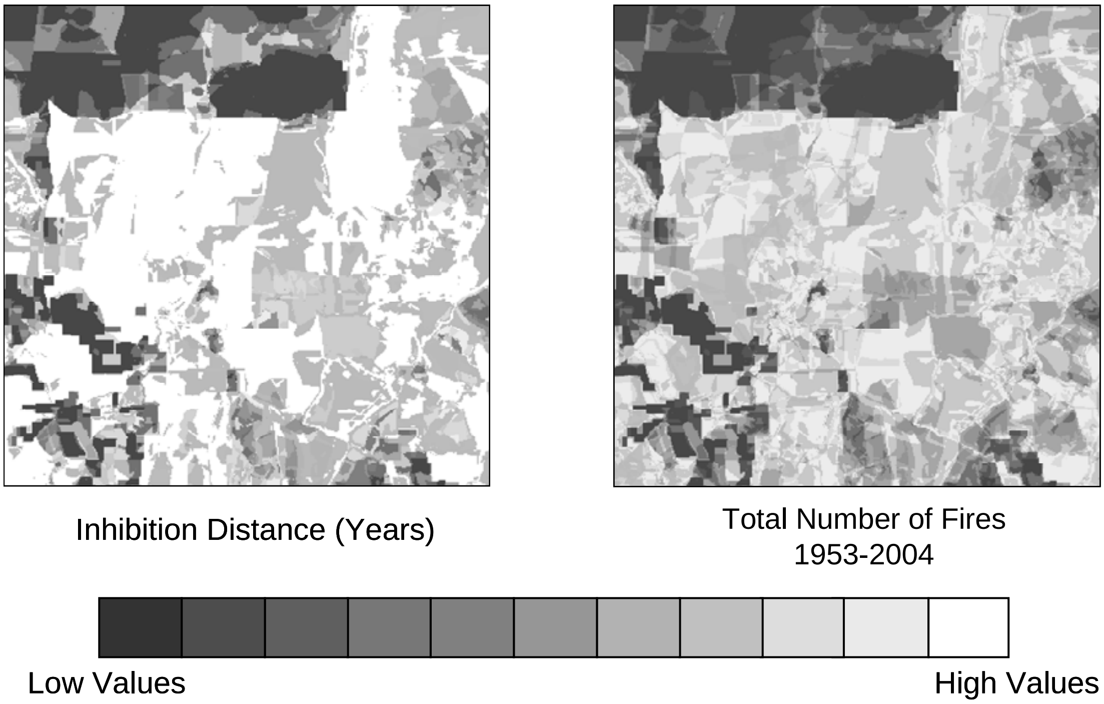

Figure 3. Spatial distribution of (a) the inhibition distance ($I_{0.5}$) and (b) fire frequency for the 40 km x 40 km trial area in the Warren Region.

Table 1: Regression of $I_{0.5}$ on ecological covariates

variable	Df	P-value	Type	Interpretation
aspect^2	1	<0.0001	- ve	Stronger inhibitory effects on southern slopes
elevation	1	<0.0001	+ve	Stronger inhibitory effects at higher elevations
slope	1	<0.0001	- ve	Stronger inhibitory effects on gently sloping sites
wetness	1	<0.0001	+ve	Stronger inhibitory effects at wet sites
veg. type	60	<0.0001		Fuel age effects differ between vegetation types

Model: $I_{0.5}$ ~ aspect^2 + elevation + slope + wetness index + vegetation type

The quadratic term for aspect is included as aspect is measured in radians, with $0 = 2\pi$, to give an indication of the general significance of the variable. Elevation, slope and aspect were derived from the ~90 m resolution SRTM digital elevation model (e.g. Berry et al. 2007); wetness index according to Beven and Kirkby (1979); and vegetation complex according to Mattiske and Havel (1998).

We have introduced I_α as a nonparametric measure of the inhibition effected by a fire over consequent fires in comparison to a perfectly random pattern. Nonparametric estimators obviate the need for making distributional assumptions, and censoring due to a limited observation period is more readily accounted for than in models with several parameters. Some of our other work includes applying functional principal components to classify the Kaplan-Meier estimator $\hat{F}_i(t)$ of the linear contact distribution for every pixel i in a map (Illian et al. 2006; Ramsay and Silverman 2006). This classification enabled the selection of experimental sites within the Warren Region based not only on homogeneous slope, aspect and wetness, but also on similarly patterned fire regimes as observed in the historical database.

Remote sensing of burn severity

Remote sensing is the most practical method available to managers of fire-prone environments for quantifying and mapping burn severity at landscape scales. The differenced Normalised Burn Ratio (ΔNBR) (Lopez Garcia and Caselles 1991; Key and Benson 2006) is among the most widely used spectral indices for the mapping of burn severity but its interpretation in terms of fire-related changes in key biophysical attributes and processes is problematic (Lentile et al. 2006). This makes it difficult to compare burn severity assessments from different sites within a single fire, at the same site in consecutive fires, or to study burn severity patterns across different forest types or environments. However, if we are to incorporate burn severity as an additional fire attribute in the proposed method for the characterisation of fire regimes it needs to be quantified using objective measurements and a well-defined biophysical scale.

For forests, we propose to quantify burn severity as a fire-induced change in leaf area index (ΔLAI). Our focus on ΔLAI is underpinned by several considerations. Firstly, LAI is a clearly defined biophysical vegetation attribute that can be objectively measured in the field (Breda 2003; Macfarlane et al. 2007) as well as by remote sensing (e.g., Baret and Guyot 1991; Peddle et al. 1999; Turner et al. 1999; Gascon et al. 2004). Secondly, the magnitude of change in forest LAI is indicative of flame lengths and scorch heights during the pass of the fire, which in turn are a good proxy for fire intensity in a given forest type (Byram 1959; Van Wagner 1973; Gould et al. 1997). Thirdly, focusing on ΔLAI for the quantification of burn severity is justified from a spectral point of view because the differences in spectral properties of pre- and post-fire forest stands are caused to a large degree by a reduction in LAI (Chuvieco et al. 2006).

Burn severity mapping in the Perth Hills area

The proposed method was developed and tested in the northern Jarrah forest of SW West Australia, focusing on the mapping of burn severity in the ~27,000 ha that burnt in the Perth Hills during the January 2005 wildfire (Cheney 2007). An empirical model for the prediction of pre-and post-fire LAI was developed from georeferenced field measurements of LAI in 40 m x 40 m plots (n=26) and spectral vegetation indices measured from Landsat Thematic Mapper imagery. A linear model of LAI as a function of the Simple Ratio (SR, see Jordan 1969) explained 86% of the observed variation in LAI. The LAI~SR model was applied with TM imagery from 2004 and 2005 to predict pre- and post-fire LAI, and their difference (ΔLAI). Confidence bounds for ΔLAI at each pixel location were obtained by bootstrapping. The lower (2.5%) confidence bound of ΔLAI was used to identify the area fire-affected by the January 2005 wildfire.

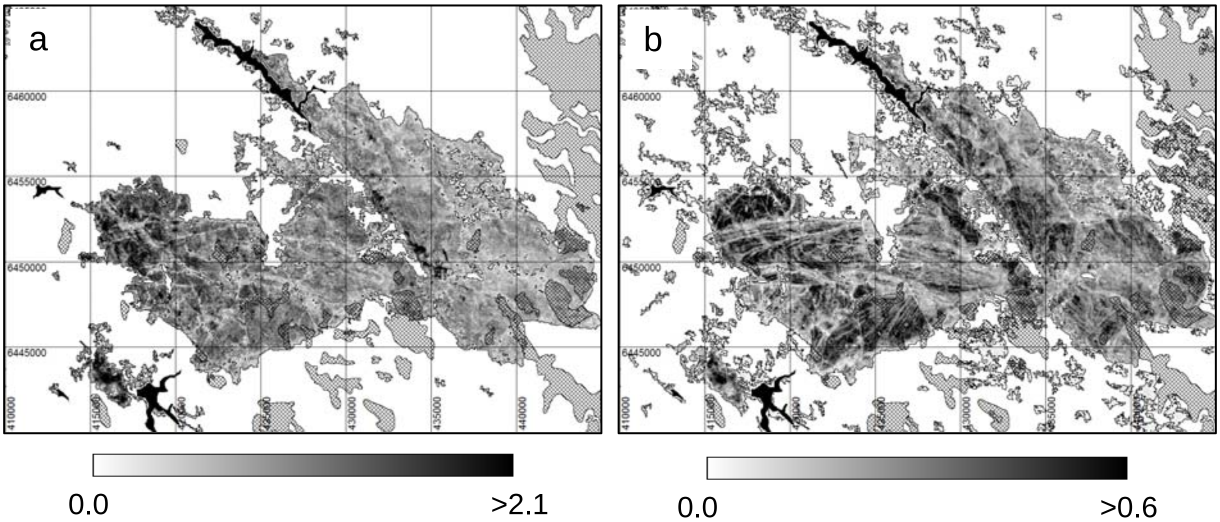

Figure 4. Remotely sensed burn severity in the area burnt during the January 2005 Perth Hills wildfire. Burn severity was computed from the pre- to post-fire difference in predicted LAI (a) or NBR (b). Hatched polygons indicate areas not included in the jarrah forest category. Reservoirs shown in black. UTM-50S gridlines at 5 km spacing.

The landscape pattern of burn severity mapped by ΔLAI (Fig. 4a) compares well with that obtained using ΔNBR (Fig. 4b) and with the mapping of burn severity by the WA Department of Environment and Conservation (Cheney 2007). ΔLAI has an important advantage over existing indices such as ΔNBR by measuring burn severity on a well-defined and universally recognised scale. This allows burn severity assessments to be compared across space and time, which is essential for incorporating burn severity into a statistical characterisation of fire regimes. As ΔLAI is readily interpreted by ecologists, hydrologists and scientists from other disciplines not directly involved in the regional management of fires, mapping burn severity as ΔLAI also allows for a more effective integration of fire impact data and other ecological information required for science based management of fire-prone forest landscapes.

Discussion

We have developed a nonparametric measure to characterise the historical pattern of fire incidence, and have trialled a method for quantifying burn severity from remotely sensed changes in LAI. Applying measures of patterning in the incidence and severity of fires are a critical first step towards a quantitative characterisation of the potential interactions between these and other attributes of a fire regime. A quantitative understanding of these interactions is key to tailoring regimes of prescribed burns to meet specific management objects, and to provide some measure of confidence in our ability in achieving these objectives, given the highly stochastic patterning of wild fire.

A key qualification in the analysis of the Warren Region data is its exclusion of prescribed burning as a factor explaining the incidence of wildfire, which represented more than 80% of the total area burnt between 1953 and 2004. A representation of the renewal function that relates purely to wildfire was not arrived at, and correlations between the $I_{0.5}$ metric and geographical variables may be confounded by the high rate of prescribed burning. For instance, the wetness index may be correlated to large $I_{0.5}$ values simply because prescribed burning were planned in this region at regular intervals to reduce fuel loads, as regeneration rates were greater due to a greater moisture availability (Didelez 2008). While such assertions remain conjecture until a more rigorous testing framework is developed, the inclusion of multi-type point processes in analysing the role of prescribed fire in inhibiting wildfire may be readily undertaken, due to the wealth of literature on marked point processes (e.g. Cox and Isham 1980). In a nonparametric setting, interactions between different types of processes may be examined through pair correlation functions related to the empty space function (Eq. 1).

Prescribed burning may instead be mapped in detail through the mapping of burn severity as demonstrated here, as prescribed fires tend to be of less severe than wildfire due to the controlled and planned implementation. The severity of previous fires at a pixel location may now be considered explicitly as the ‘marks’, or covariates, associated with each occurrence of fire. Estimates of the “waiting time” at a location until the next occurrence of wildfire may then be modelled on burn severity, informing management as to the efficacy of prescribed burns in mitigating the occurrence and severity of wildfire. The availability of the current Landsat archive (1988-2007) will likely enable the retrospective mapping of burn severity across the forests and woodlands of southern Australia. Where historical fire records are available, then the incorporation of severity information into models of fire occurrence will provide more accurate characterisations of the fire regime.

We conclude by speculating how marked point processes may contribute to the optimal design of prescribed burn regimes. A more general class of marked processes are random closed sets, where the marks of a spatial point process define the shape, size, clustering and timing of prescribed burn patches, and the consequent accumulation of fuels over time. Similar models have been employed in describing endocytic events of *in vivo* cells (Ayala et al. 2006). Working with these models we envisage the design of prescribed burn regimes through the employment of adaptive optimisers to determine a space-time “geometry” that maximises a manager’s objective function (e.g. minimise infrastructure risk from wildfire while maximising the heterogeneity of fuel age patterns). Such an optimiser may then be coupled to real landscapes (e.g. current patterns of ignition, housing and topographical features) and a fire simulator to ideate the optimal regime of prescribed burns for a particular locale when constrained to a set budget.

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